X. "On the Power of the Eye and the Microscope to see Parallel Lines." By J. A. Broun, F.R.S. Received June 16, 1875.

Since Nobert's test-lines have been employed for the purpose of determining the comparative powers of microscopes, several curious speculations have been made and conclusions arrived at by different well-known microscopists as to the ultimate capability of that instrument and even as to the ultimate atoms of matter. The lines in Nobert's test-bands were believed to approach to each other in a regularly diminishing series of distances, such that when the intervals left by the graving-point were about $\frac{1}{80,000}$ of an English inch wide, no microscope, however high the theoretical power, could show the separation of the lines.

The Jury of the Great Exhibition of 1851, in the Report (p. 268), said that to see the lines in the first and second bands of a Nobert's test-plate of 10 bands a power of 100 was sufficient, whereas to distinguish those of the tenth band a magnifying-power of 2000 was required. Dr. Lardner (in his Museum of Science and Art) considered this assertion erroneous, and stated that if a power of 100 could show the lines when there were 11,000 to the inch (as in the first band), a power of 450 should show the lines when there were 50,000 to the inch (as in the tenth band). As there can be no doubt of the accuracy of the Jury's statement, there has been considerable difficulty in ascertaining the cause of the great difference betwixt fact and theory; and the most absurd hypotheses have been suggested for this end. This and the speculations of different microscopists induced me several years ago to begin the examination of this question.

The first thing requiring to be clearly understood was, what the microscopes were employed to examine, what Nobert's test-bands really were. Some microscopists stated the number of lines to the inch when they could not be seen to be about 80,000; whereas Dr. Carpenter (in his work on the Microscope and its Revelations) said "it was a matter of faith whether lines existed at a narrower interval than $\frac{1}{84,000}$ of an inch." In this case if the lines were as wide as the intervals, the number to the inch would be 42,000. It was then essential to know what were the widths of the lines and intervals in the different bands, whether as the intervals would appear equally light, the lines were equally dark; and, finally, with reference to the bands in which the lines were so close that they could not be seen, whether there really were lines at all, or lines which really ought to be seen were the microscope practically equal to its theoretical power—whether, in fact, the failure was not rather in the engraving-machine than in the microscope.

It was only in 1869 that I was able, through the kindness of Mr. Eulenstein, of Dresden (previously of Cannstadt), to examine the magni-

fied photographs of Nobert's test-bands by Dr. E. Carter, Surgeon of the U.S. Army. The results of this examination will be given at the end of this note.

I was induced meanwhile to examine the power of the eye, in order to compare it with the power of the microscope, and to determine what a microscope of given power should be able to show. The following observations had been made early in 1869, before I was acquainted with the observations of Dr. Jurin and of Tobias Mayer, to which I shall allude.

The first question which presented itself was as to the power of the eye to see single lines under the ordinary illumination of a northern sky.

1*st observation.*—A black line 0·042 inch wide, 1·75 inch long, drawn with common writing-ink on white paper, and a white line of the same width and length between two black lines, each 0·20 inch wide, were seen equally well within a room lighted by a window to N.W. at a distance of 30 feet, the angle subtended by the width of the lines being 24″ nearly.

2*nd observation.*—A dark-brown hair, 0·0026 inch wide, 2·5 inches long, was fixed by dots of transparent gum-arabic to the window-pane, and was seen against the N.W. sky by a young eye at 36 feet (I could not see it myself at a greater distance than 30 feet): the diameter of the hair subtended an angle of 1″·24 at the eye. The same eye examined fine lines divided on glass at a distance of 6 inches, and, other things equal, should have been able to see a line $\frac{1}{27,700}$ inch wide at that distance. [June 5, 1875. I find that a young eye can see lines on glass $\frac{1}{10,000}$ inch wide, $\frac{1}{28}$ inch long, angle 3″·5 nearly.]

Dr. Jurin could see a silver wire $\frac{1}{485}$ inch diameter placed on white paper when the diameter subtended an angle of 3″·5, and a silk fibre one fourth the diameter of the wire when it subtended an angle of 3″·35*.

3*rd observation.*—Whether the length of the line affects its visibility. The hair just observed was cut into pieces of different lengths and fixed, as before, to the window-pane; they could be seen at the following distances:—

Length of hair.	Distance seen.	Angle subtended by	
		Diameter.	Length.
in.	feet.	″	″
0·90	37	1·21	413
0·25	32	1·39	134
0·133	22	2·03	104
0·020	10	4·46	86

* See Jurin's essay on distinct and indistinct vision in Smith's 'Complete System of Opticks,' 1738. I am acquainted with Jurin's observations from the Rev. Father Pezenas's translation of Smith's work, 'Cours complet d'Optique,' 1767, p. 282.

The hair 0·9 inch long was one foot further off than that 2·5 inch long on the day preceding (2nd observation). The difference was due, partly at least, to the different light of the sky.

4th observation.—The previous observation shows that the line is seen at a greater distance as the length increases till a limiting angle is attained, after which increase of length has no effect on the visibility. The following observations were made to determine approximately the law which relates visibility to length.

Lines of different lengths, 0·045 inch wide, were drawn on different slips of white paper (5·8 by 4·5 inches); the papers were pinned successively to a plank placed vertically in the shade (out of doors) with a clear sky (April, 6 P.M.); the mean distance of disappearance on retiring and of appearance on approaching the lines was taken.

Length.	Distance.	Angle subtended by			$\alpha \sqrt[3]{\beta}$.
		Length β.	Width α.		
			Observed.	Calculated.	
in.	feet.	$''$	$''$	$''$	
0·045	......	......	......	26·4	
0·125	53	41	14·6	14·5	50
0·245	68	62	11·4	11·2	45
0·470	84	96	9·2	9·2	42
0·970	100	167	7·7	7·7	42
1·800	114	272	6·8	6·8	44
3·400	129	453	6·0	6·0	46

It will be seen that as the lengths of the lines increase in a geometrical progression (nearly), the distances increase in an arithmetical progression (nearly). It has been easy then to represent the observed angles (α) by the following equation:—

$$\alpha'' = \frac{14·5}{\log l - 1·10}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)^*$$

where α is expressed in seconds of arc, and l, the length of the line, is in units of 0·001 inch. The calculated values agree very nearly with those observed. The angle for a square of 0·045 inch calculated is 26″·4, which is very near to the value observed on a previous occasion. α (or $\tan \alpha$) becomes infinite for l=12·6 (0·0126 inch); but the formula does not hold for lines in which the length is less than the width. These belong to another case, that in which the lengths of the lines are constant and the width variable.

Having examined the power of the eye to see single lines, I now sought how this power would be affected when more lines than one were placed parallel to each other, and with intervals equal to their widths.

* This equation may be put under the following form, where D, the distance of the observer, and l are in units of one inch:—

$$D = 642 \log 79·4\, l.$$

5th observation.—Four dark-brown hairs having been arranged on paper at equal intervals by means of modeller's wax attached to the ends, and then fixed at the ends with gum-arabic, the paper was cut away between the ends, and the two slips of paper to which the ends were gummed were fixed to the window-pane: the diameter of the hairs was $\frac{1}{375}$ inch (0·00267) very nearly, the lengths were nearly 1 inch, and the intervals were very nearly the same as the diameter of the hair, the whole width being 0·019 inch. The hairs could be seen to be more than one at 28 inches distance, and they could be counted at 21 inches; the angles subtended by the intervals at these distances were $20''$ and $26''{\cdot}5$ respectively.

It thus appears that at a distance greater than 28 inches the four hairs appeared as one, when each hair subtended an angle sixteen times greater than it could be observed at when seen alone (see the 3rd observation). This curious fact was pointed out by Dr. Jurin in the essay already cited. He found when two pins were placed near to each other on a window, that the interval between them could not be perceived when it subtended an angle of $40''$, whereas a single pin could be seen at an angle of from 2 to 3 seconds. Mayer, who also made observations on parallel lines nearly twenty years after Jurin, does not seem to have remarked this fact*.

The 5th observation was repeated with four white hairs from a horse's tail; these were arranged at equal intervals, the mean diameter of the hair being 0·0105 and the mean interval 0·0110 inch. The hairs could be seen to be more than one 9 feet distant, when the angle subtended by each hair was $23''{\cdot}1$; and the hairs could be counted at 6 feet distant, when the angle was $30''{\cdot}7$. These angles are about one sixth greater than for the human hairs, the difference being probably due to the difference of light, and perhaps partly to the different length, which was not noted†.

6th observation.—A series of lines 0·7 inch long were drawn on separate slips of paper with different widths and intervals. The papers were fixed successively to the wall of a room lighted by a window to N.W., the light falling at an angle of about 45° on the paper. The following Table contains the results of the observations, first, when the intervals and lines were of equal width, and, second, when the intervals were 1, 2, 3, . . . times the width of the lines.

* Mayer's observations are given in Pezenas's translation of Smith's 'Optics,' t. ii. p. 409. I am not acquainted with the original memoir. As already stated, Jurin's and Mayer's observations were known to me only after the above observations had been made.

† Dr. Jurin has given, as an example of the difficulty of counting parallel lines, the following series:—

and has shown the advantage of employing commas in such numbers as the following, 100000000000 and 100,000,000,000.

Lines and intervals equal.			Lines and intervals unequal.			
Mean width.	Distance.	Angle.	Width of Lines.	Width of Intervals.	Distance.	Angle.
in.	feet.	″	in.	in.	feet.	″
0·021	8·3	44	0·041	0·021	9·5	37·0
0·041	15·5	45	0·045	0·079	21·7	38·0
0·081	25·0	56	0·043	0·121	28·5	26·0
0·118	36·7	55	0·044	0·164	34·0	22·0
0·164	47·5	59	0·044	0·203	39·0	19·4
			0·045	0·241	40·5	19·0
			0·044	0·477	55·5	13·5

When the lines and intervals were equal, the angle increased from the smallest (0·021 inch) to the largest (0·164 inch); the greater angle for the middle width (0·081) is probably connected with some irregularity in the lines; the increase of the angle at the greater distance is probably chiefly due to the constancy of the length of the lines (see 8th observation).

When the lines and intervals are unequal, the angle subtended by the width of the line diminishes as the interval increases; the limiting angle would be that at which a *single* line 0·70 inch long and 0·044 inch wide would disappear.

7th observation.—It was now sought to determine in what degree the angle of visibility for parallel lines varied with the relative darkness of the lines, the intervals between the lines remaining equally bright. One drop of a weak writing-ink having been mixed with thirty drops of water, four lines, 0·70 inch long, 0·081 inch wide, were made (at intervals of 0·081 inch) with a camel-hair pencil; the first set (I_1) received only one coat, the second (I_2) two coats, and I_5 five coats. The slips of paper having been pinned successively to the wall of the room as before, the following are the results of the observations:—

Tint.	Distance.	Angle.	Calculated.
	feet.	″	″
I_1	15·5	90·0	91·0
I_2	19·5	71·5	69·7
I_3	21·5	64·7	63·1
I_4	23·0	60·5	60·0
I_5	23·6	59·0	58·4
⋮	⋮	⋮	⋮
I_n	25·0	55·7	56·0

The last (I_n) is the observation already given (6th observation) for the same lines made with a drawing-pen and as black as they could be made. It was found that one hundred coats of watered ink did not make lines so dark as those of I_n; and it is obvious that increase of darkness after the fifth coat (a very faint shade) made little difference in the visibility of the

lines. On the other hand, the tint may be made so faint as to be imperceptible at 6 inches (the distance for a young eye of faint or small objects). The preceding observations are represented nearly by the following formula:—

$$\alpha - 56 = \frac{19 \cdot 2}{1 \cdot 55^t - 1}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where α is the smallest angle (in seconds) at which the separation of the lines was visible, and t is the *tint* or number of the coats of watered ink. The calculated values are given in the Table.

When $t = \frac{1}{46}$ (of the first tint) the lines, according to the formula, should have been just visible at 8 inches from the eye, or a weaker shade on white paper than that made by one drop of the first tint with forty-six drops of water could not have been seen. On the other hand, when $t = 12$, the difference of the angle of visibility from that for absolute blackness is only $0'' \cdot 1$. The constants in this and the other formulæ will depend of course on various circumstances of illumination, the state of the individual eye, &c.

Mayer made a series of observations with several parallel lines drawn with China ink on white paper (well stretched), the width of the lines and intervals in one case being 0·032 of an English inch (0·36 *de ligne*). These lines he could perceive to be several at 11 feet (*pieds de Roi*) distance with the light from an open window to north, or when the angle subtended by the interval was 47″. He then made observations with the same lines lighted by a wax candle placed at different distances from them.

I give Mayer's observations for this set of lines here for comparison with the preceding results for different tints.

D.	d.	α. Observed.	Angle α, calculated by (3).	Error.	(5).	Error.	(6).	Error.
feet.	feet.	″	″	″	″	″	″	″
7·47	0·5	69	63	− 6	66	−3	69	0
6·53	1·0	79	79	0	78	0	79	0
5·73	2·0	90	99	+ 9	96	+6	92	+2
4·73	3·0	109	114	+ 5	108	−1	103	−6
4·48	4·0	115	125	+10	118	+3	112	−3
3·51	8·0	147	158	+11	146	−1	141	−6
3·00	13·0	172	185	+15	172	0	172	0

Mayer considered that he explained his result by supposing the limiting angle (α) of distinct vision to be as the cube root of the distance of the candle from the paper, or

$$\alpha = 79 \sqrt[3]{d}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where α is in seconds and d is in *pieds de Roi*. He also arrived at the following curious conclusion. Since the limiting value of α for the

same lines in full daylight was 47″, by substituting this value in equation (3), he found $d=0{\cdot}2$ foot; and "we may conclude," he says, "that the light of day is as strong as that of a candle at one fifth of a foot from the object. Consequently if we wish to light an object with a candle as strongly as it would be by daylight [and even by strong sunlight, as Mayer found], we must employ twenty-five lighted candles placed at a distance of one foot from the object"*!

Had Mayer's formula been an exact representation of the observations, he might have concluded that the eye could separate parallel lines as well with twenty-five candles at one foot distance as with full sunlight; but it will be seen that the errors of the formula increase as d increases and diminishes. An equation of the form given by Mayer which best represents the observations is found by least squares to be

$$a=77{\cdot}6\, d^{\frac{1}{2{\cdot}57}}; \ldots\ldots\ldots\ldots (4)$$

but this also fails when d is small. The following equation best represents Mayer's observations, including that for daylight:—

$$a-47=25\sqrt{d}+30\log(d+0{\cdot}9). \ldots\ldots\ldots\ldots (5)$$

When $d=0$, $a=48''{\cdot}3$. The errors of this equation are given in the preceding Table. But it will be seen here also, from the distances D of the observer (which I have calculated from the angles given by Mayer), that when the distances d of the candle increase in a geometrical progression, the distances D diminish in an arithmetical progression as nearly as the accuracy of the observations admit. This fact I have myself verified by repeating Mayer's observations. We may then represent the observations by an equation of the same form as the others. The following has its computed values and errors given in the Table:—

$$a=\frac{79}{1-0{\cdot}485\log d} \ldots\ldots\ldots\ldots (6)\dagger$$

It follows from this formula that when the candle is 140 feet from the paper, the eye at 8 inches from it could just see the lines and spaces; when $d=\frac{1}{25}$ foot, $a=47''$, the smallest angle under which the lines and spaces can be seen.

It might have been supposed that the distance of the observer from the paper would vary inversely as the illumination, or that a should vary as d^2, which, it will be seen, is very far from being the case.

The 7th observation previously given represents more nearly the case of the examination of test-lines on glass, the spaces being equally bright, or nearly so, in all cases, while the lines have a variable depth of shade. In Mayer's observations both spaces and lines receive less light as the candle is removed. The impression on the retina for the separation of

* Pezenas, 'Cours complet d'Optique,' t. ii. p. 415.

† Or, $D=6{\cdot}53\,(1-0{\cdot}485\log d)$.

the lines receives no improvement in the first case by any increase of depth of tint of the lines beyond a certain feeble shade, nor in the second (Mayer's) case by any increase of the illumination of both lines and spaces beyond that of a candle held near.

8th observation.—It was sought whether the visibility of parallel lines increased with their length, as in the case of single lines. Four long parallel lines having been drawn, of the width and at the interval of 0·048 inch, on a sheet of white paper, this was pinned, as before, to a smooth plank and placed in the open air in the shade (before sunset); the lines were covered by a sheet of the same paper so as to show variable lengths. The following Table contains the results of the observations :—

l.	D.	α.	(7).	Errors.	β.
in.	ft. in.				
0·4	16 8	$49''\cdot5$	$49''\cdot5$	$0''\cdot0$	$413''$
0·8	17 4	47·6	47·4	−0·2	794
1·6	18 5	44·8	45·5	+0·7	1490
3·2	18 10	43·8	43·8	0·0	2921
6·4	19 5	42·5	42·1	−0·4	5664

D is the distance of the observer, l the length of the lines, β is the angle subtended by the length at the eye of the observer; α, the angle subtended by the width of the lines, is represented nearly by the following formula :—

$$\alpha = \frac{352}{\log l + 4\cdot53}, \qquad (7)$$

where l, the length of the lines, is expressed in units of 0·001 inch. The variation of the angle is comparatively small in this case. The law of variation of α seems to change when the length of the lines becomes less than the width of the whole.

9th observation.—The following observations were made with short parallel lines drawn *separately* on the same sheet, with the same width of intervals and lines as in the last case :—

l.	D.	α.	(8).	Errors.	β.	$\alpha\sqrt[3]{\beta}$.
in.	in.					
0·40	204	$48''\cdot6$	$48''\cdot0$	$-0''\cdot6$	$404''$	359
0·20	186	53·2	53·5	+0·3	222	322
0·10	168	58·9	60·6	+1·7	123	293
0·05	144	68·8	69·9	+1·1	72	286
0·025	120	82·5	82·5	0·0	43	289
0·012	96	103·1	101·0	−2·1	26	306

The observations are represented nearly by the equation

$$\alpha = \frac{137}{\log l + 0\cdot27}. \qquad (8)$$

It will be seen tha tthe angle α increases rapidly as the length dimi-

nishes; on the other hand, β, the angle subtended by the length of the lines, decreases much more rapidly. In this case, as in that of the 4th observation, we find α and β are connected nearly by the following formula:—

$$\alpha \sqrt[3]{\beta} = \text{constant} \ldots\ldots\ldots\ldots (9)$$

10*th observation.*—The long parallel lines were seen by different persons at a greater distance when inclined to the horizontal by an angle of about 25° below it to the right and above it to the left, but the visibility varies at different angles for different persons.

11*th observation.*—Jurin's observations of the difference of visibility of parallel lines and a single line had reference to the case of only two black lines with a white line between. On comparing the distances at which parallel lines could be seen 0·4 inch long and of the same width and interval as before (0·048 inch), it was found that it was most difficult to separate the lines when there were only two. The following are the observations made:—

Number of lines.	D. ft.	D. in.	α. ″
1	50	0	16
2	13	4	62
3	15	0	55
4	17	2	48

As my object at present has been simply to state the facts observed, I shall now proceed to my examination of the photographs of Nobert's test-bands. The use of these I owed to the kindness of Mr. Eulenstein (a histologist well acquainted with the microscope and its tests), who informed me that these lines were made with the same diamond-point, but that the pressure on the point is made to increase as the number of lines to the inch diminishes. The widths of the lines and of their interspaces were measured by me by means of a glass scale accurately divided with one thousand divisions to the inch, the dividing lines being about one tenth the width of the interspaces. This scale was placed one half on thin white paper, the other covering the photographed lines. The readings were made with the aid of a pocket-lens of 1·1 inch focal distance. The width of every line and interspace was measured for the first six bands; from eight to ten lines and spaces were measured from the VII.th to the XIII.th bands inclusive; in each case the means of these measures are given. For the remaining bands the number of lines and their spaces were counted and their whole width observed to the XVII.th band. In the XVIII.th and XIX.th bands the lines run into each other in different places (in some of the previous bands the lines occasionally fail). It was evident that for the highest bands the machine failed to make the number of separate lines which were drawn. In several cases diffraction-fringes interfered with the accuracy of the measures; but as these were generally made

from bands near the middle of the photographs, there could be no doubt whether the line was a fringe or not.

The following Table contains the results of these observations :—

Measures of Nobert's Test-lines.

Band.	Number of lines.	Width of			Number to the inch.			Ratio.
		Line.	Space.	Band.	Lines.	Spaces.	Both.	
I.	7	28·57	58·67	553	35,000	17,040	11,460	1·00
II.	10	35·10	23·00	560	28,500	43,450	17,210	1·68
III.	13	27·85	14·85	555	35,910	67,340	23,420	2·11
IV.	15	19·13	15·57	505	52,270	64,230	28,820	3·10
V.	17	14·10	15·20	480	71,430	65,790	34,250	3·87
VI.	20	11·55	13·90	495	86,580	71,940	39,290	4·23
VII.	23	12·64	9·14	505	79,110	109,410	45,910	4·65
VIII.	25	8·71	10·71	475	114,810	93,370	51,390	5·49
IX.	28	7·47	9·93	478	133,870	100,700	57,470	5·92
X.	30	8·11	7·44	460	123,300	134,410	64,310	7·25
XI.	34	8·10	6·90	500	123,300	144,930	66,670	7·25
XII.	37	6·81	7·28	503	146,840	137,360	70,970	8·08
XIII.	40	6·62	6·00	500	151,060	166,670	79,240	8·89
XIV.	43	6·00	6·00	510	166,670	166,670	83,330	9·81
XV.	45	5·56	5·56	495	179,860	179,860	89,930	10·50
XVI.	(40)	......	......	522	....	....	77,280	
XVII.	(40)	......	......	{503 / 512}	....	....	77,880	
XVIII.	(40)	......	......	{489 / 511}	....	....	79,240	
XIX.	(40)	......	......	540	....	....	75,760?	

Notes.—These measures are frequently mere approximations; and in several bands the graving-point has made a wonderful approach to an equality of width of lines and spaces; indeed these lines are marvels of mechanical skill. If, in the case of each band, the first and last lines had been drawn longer than the rest, it would have been possible to measure the width of a line with considerable accuracy, since, as has been shown, the visibility of a single line is nearly twenty times that for the series.

The widths of the lines and spaces are those taken from the photographs, the unit being $\frac{1}{1000}$ inch. The photographs are magnified to 1000 times. In bands XVII. and XVIII. second measures are given from photographs magnifying to 1600 times (but reduced to the same unit). The number of lines in ( ) are the numbers counted for which the total width was measured. The number for the XIX.th is deduced from the measure of a few where the lines were most distinct. The numbers of lines and spaces to an inch are the numbers which could be put in an inch laid side by side (without interval). Under " Both " is given the number of lines to the inch (with interspaces), as in the bands. The "Ratio" is that of the number for the widest space (17,000 to the inch) to the number for the widest line or space in the following bands.

It will be seen that the least width of the lines which can be counted and measured on the photographs is about $\frac{1}{160,000}$ of an inch (XIII.th band). We have seen (5th observation) that dark parallel lines on glass can be seen with transmitted light when their width subtends an angle of 20″ to 26″; so that lines stopping the light moderately (7th observation) of $\frac{1}{160,00}$-

of an inch wide should be seen with a power of 125, and counted with a power of 160 (the distance for the unaided eye being considered 8 inches). We have, however, obviously in the high bands to include the case of observation 7, the lines on the photographs being excessively faint. When we add to this fact (a most important one when such lines are supposed to give some measure of the power of the microscope) that it appears that separate lines cannot be drawn of a less width than about $\frac{1}{180,000}$ of an inch under the diminished pressure of Mr. Nobert's machine without the graving-point sliding into previous grooves, we have a sufficient explanation why the power of the microscope cannot be measured by these lines.

The following are the conclusions of this note:—

1st. That lines can be seen by the naked eye with transmitted light the width of which subtends an angle of about 1″ (2nd observation).

2nd. That the visibility of a line, or the distance at which it can be seen, depends on the logarithm of its length, the product of the angle subtended by the width and the cube root of that subtended by the length being nearly constant (4th observation).

3rd. Short parallel lines could be seen by transmitted light when the angle formed by the width of the spaces and intervals was 20″ (5th observation).

4th. The visibility of lines of the same width increases as the distance between them decreases (6th observation).

5th. The visibility of parallel lines depends on the darkness of the shade or tint of the lines up to a certain feeble tint, after which no blacking of the lines increases the visibility; the distance to which the lines can be seen depends on c^t, where c is a constant and t is the number of the tint or shade (the number of coats of a weak tint) (7th observation).

6th. The visibility of dark parallel lines lighted with a candle depends on the logarithm of the distance of the candle from the lines; and they can be seen as well with a candle placed quite near as with the strongest daylight. This results from Tobias Mayer's observations.

7th. The visibility of parallel lines depends on the logarithm of their length, as in the case of single lines, the variation being much greater for short parallel lines than for long ones. Also for short parallel lines the product $\alpha\sqrt[3]{\beta}$ is nearly constant, as for single lines (see second conclusion).

8th. Parallel lines are least visible when there are only two, and increase in visibility with their number.

9th. Nobert's test-lines fail as a test for the microscope, especially in the highest bands, from the incapacity of the machine to make separate lines at less intervals and of less width than $\frac{1}{160,000}$ of an inch; they also fail, in all probability, on account of the faintness of the tint or shade of the lines made on the retina.